Sampler algorithm for non-convex inverse problem

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with the collaboration of
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The Galaxy volume

Solar systems fill $\sim 3 \cdot 10^{-10}$ of the volume of the galaxy

~~Most of the galaxy: empty!~~

Most of the galaxy: **Interstellar Medium**!

Observations of GMC: Orion B in visible frequencies

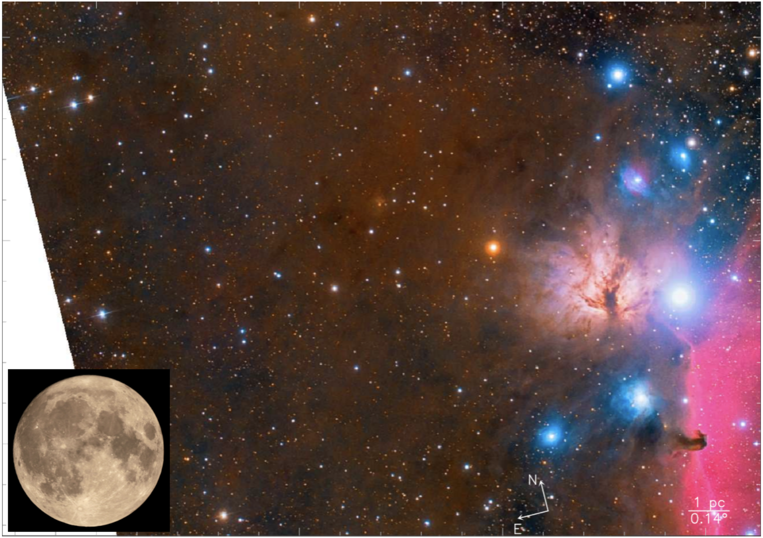

Figure: Image from Pety et al. [2016]

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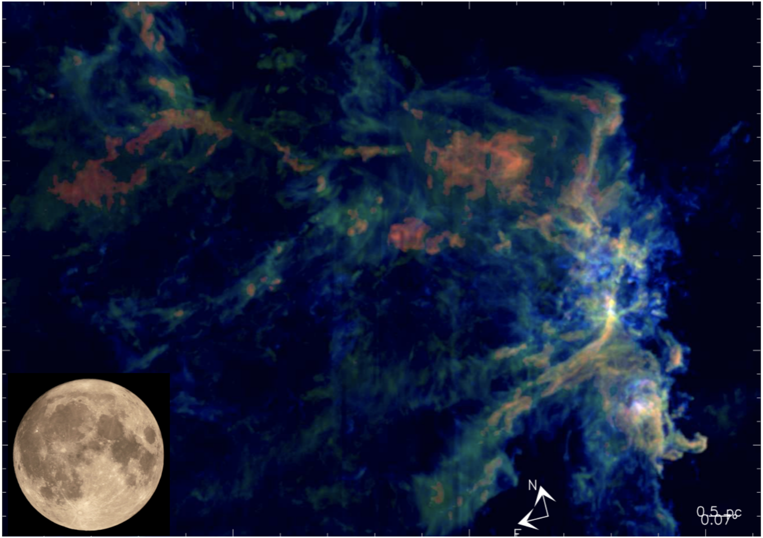

Figure: Image from Pety et al. [2016]
blue: ^{12}CO, green: ^{13}CO, red: C^{18}O

Photo-Dissociation Region (PDR)

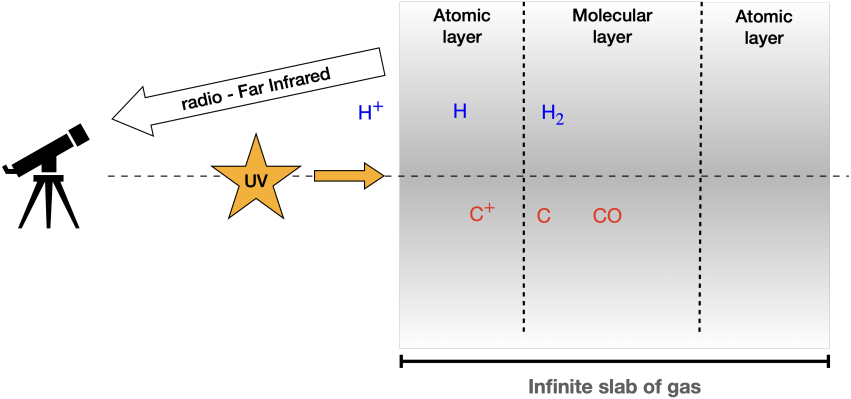

Figure: Structure of a PDR

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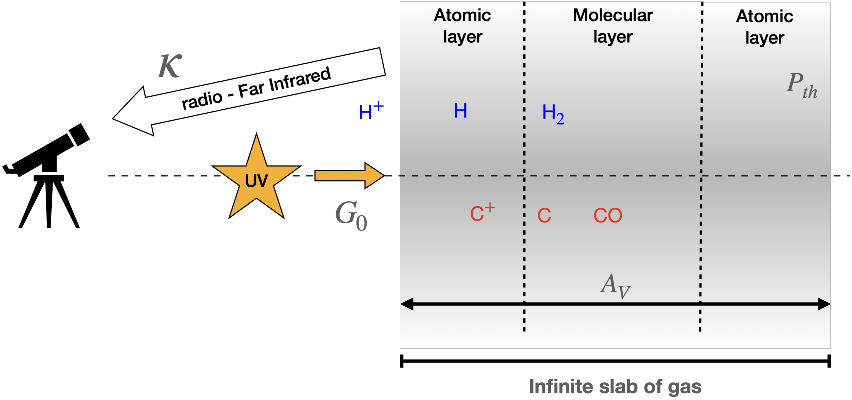
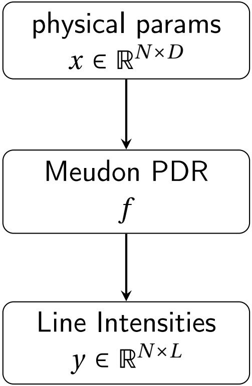


Figure: Structure of a PDR

Meudon PDR code: numerical simulation of a PDR

- Introduced in Le Petit et al. [2006].
- for stationary 1D slab of gas, solves:
  1. radiative transfer
  2. chemistry
  3. thermal balance

  that are all **coupled**!



Can we infer x from y and f?
no ground truth → with **credibility intervals**

Current state of the art in astrophysics

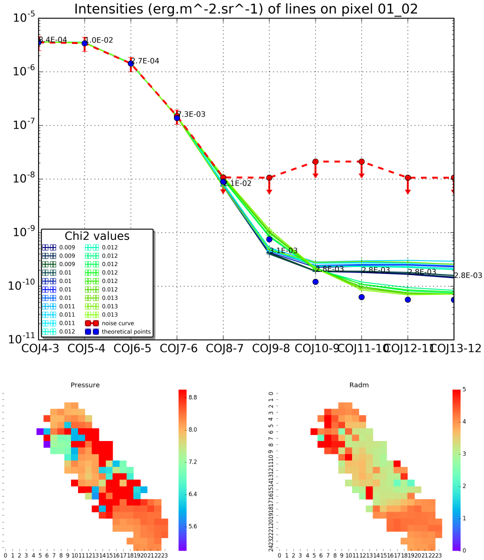


Figure: MLE maps inference

Bayesian map inversion

inference with credibility interval

$\Downarrow$

a posteriori probability distribution $\mathbb{P}\left[x \mid y\right]$

$$\underbrace{\mathbb{P}\left[x \mid y\right]}_{\textbf{a posteriori}} \propto \underbrace{\mathbb{P}\left[y \mid x\right]}_{\text{likelihood}} \times \underbrace{\mathbb{P}\left[x\right]}_{\text{a priori}}$$

Complex distribution

$\Longrightarrow$ impossible to manipulate as is

$\Longrightarrow$ sampling with MCMC

Observation model

$$\forall n, \ell,\ y_{n,\ell} = \max\left\{\omega,\ \epsilon^{(m)}_{n,\ell} f_\ell(x_n) + \epsilon^{(a)}_{n,\ell}\right\}$$

with

- $\epsilon^{(a)}_{n,\ell}$: additive noise (thermal, instruments)
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- ω : minimum detectable value by telescope

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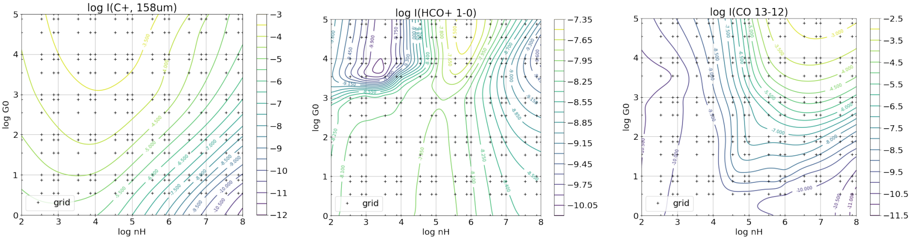

Figure: f_ℓ for some lines ℓ, for 1 pixel

Including a priori information

a priori information on x:

- spatial regularization: L2 penalty on image Laplacian
- f_ℓ: estimated from a grid
  → **constraint of belonging to a cube** (convex enveloppe of grid)
  ⚠non smooth prior ⚠but can be tempered with a smooth penalty

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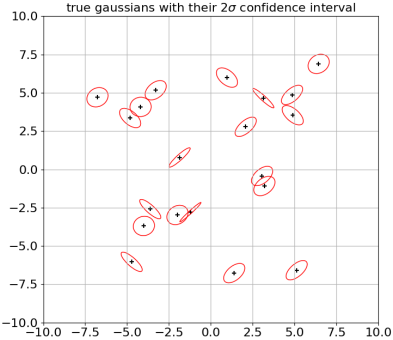
Final sampler : random combination of these two kernels

Illustration: Gaussian Mixture in a square

Illustration that our algorithm explores interesting **local minima** :

- mixture of 20 gaussians
- with constraint $x \in [-10, 10] \times [-10, 10]$



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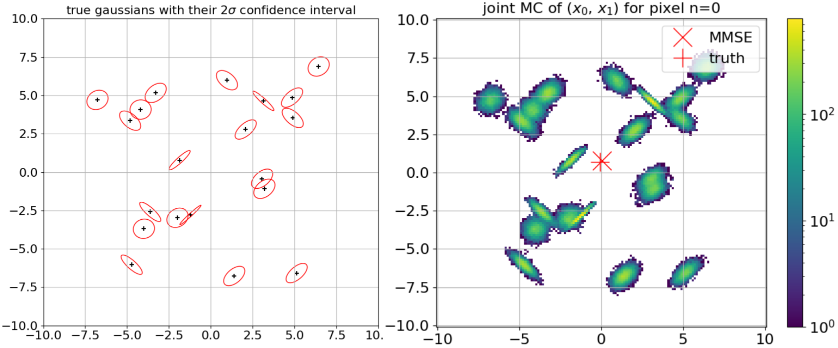


Toy case 1: Time Series Inversion

$$y_{n,\ell} = \epsilon^{(m)}_{n,\ell} f(x_n) + \epsilon^{(a)}_{n,\ell} \text{ with } f : x \in \mathbb{R} \mapsto e^x,\ \sigma_a = 1,\ \sigma_m \sim 10\%$$

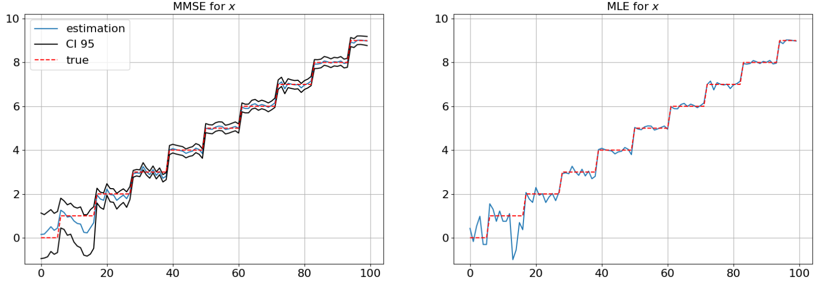
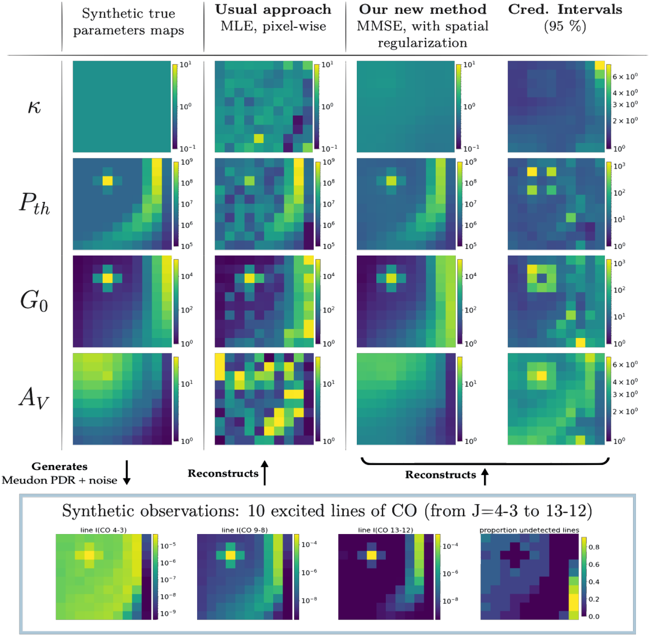


Table: Estimation Summary

estimator	MSE	SNR
MMSE	3.6	28.8
MLE	10.5	24.1

Astrophysical Toy case: Map Inversion



Application to NGC 7023 (1 pixel)

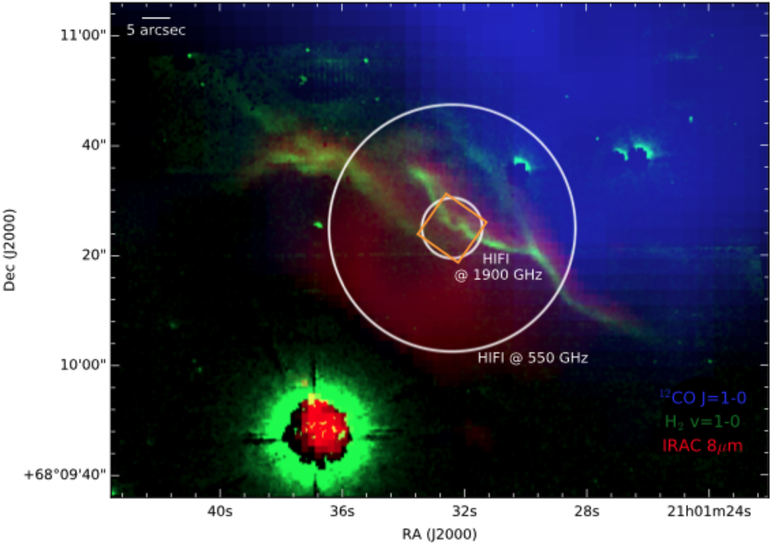


Figure: from Joblin et al. [2018]

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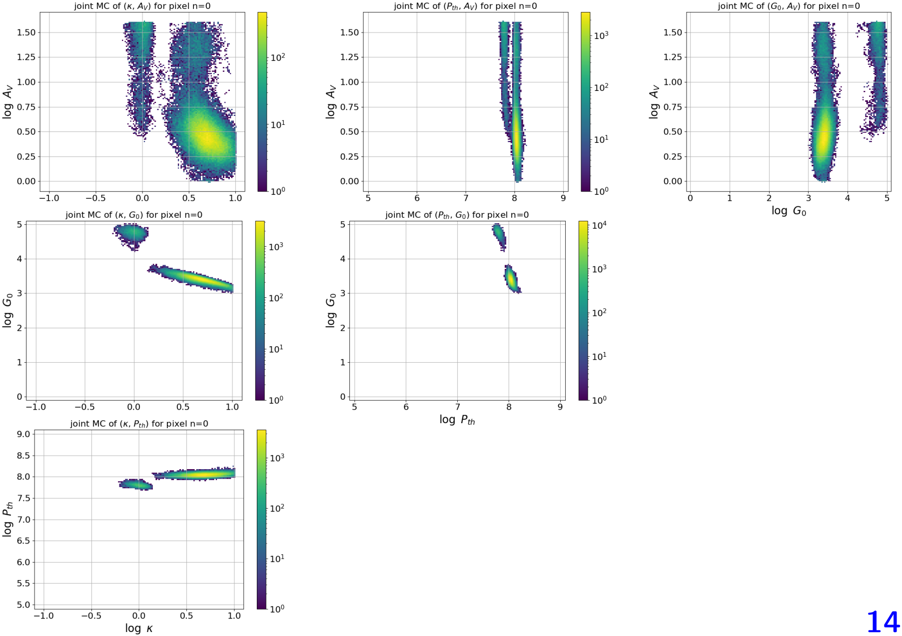

Summary

- Definition of a MCMC sampler with
  1. P-MALA kernel to tackle regularity issues
  2. MTM kernel to tackle the non-log-concavity of the posterior
- Evaluation of the method on toy data
- Application to real world data

Thank you for your attention!

C. Joblin, E. Bron, C. Pinto, P. Pilleri, F. Le Petit, M. Gerin, J. Le Bourlot, A. Fuente, O. Berne, J. R. Goicoechea, E. Habart, M. Köhler, D. Teyssier, Z. Nagy, J. Montillaud, C. Vastel, J. Cernicharo, M. Röllig, V. Ossenkopf-Okada, and E. A. Bergin. Structure of photodissociation fronts in star-forming regions revealed by *Herschel* observations of high-J CO emission lines. Astronomy & Astrophysics, 615:A129, July 2018. ISSN 0004-6361, 1432-0746. doi: 10.1051/0004-6361/201832611. URL https://www.aanda.org/10.1051/0004-6361/201832611.

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